Network Cycles and Welfare*

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Abstract

In a wide variety of social and economic settings (e.g. crime, education, political activism, technology adoption), players' returns to their efforts depend on how much effort others exert. Modeling these situations as a network game with strategic complementarities, we show that a player's *cycle centrality*—a weighted sum of the number of network cycles that she is in—determines the extent to which she benefits from her complementarities with others. In contrast to the widely-used Bonacich centrality—which measures how efforts propagate through the network—cycle centrality measures how the *variance of efforts* propagates through the network. A utilitarian social planner who can incentivize one player's effort targets the one with the highest cycle centrality.

Keywords: Strategic complementarities, social networks, Nash equilibrium, centrality, cycles, welfare.

***JEL*:** C72, D85, H41, K42, L14, O33.

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1 Introduction

> "I said that I would see you because I heard you are a serious man, to be treated with respect. But I must say no to you. And I'll give you my reasons: It's true that I have a lot of friends in politics. But they wouldn't be friendly very long if they knew that my business was drugs instead of gambling—which they consider a harmless vice. But drugs, that's a dirty business."

So says Don Corleone when Sollozzo—another criminal operating in New York City—offers him a share of the narcotics business (Puzo 1969). This situation illustrates two properties of the criminal business. First, criminals' objectives are heterogeneous: Whereas Sollozzo focuses on narcotics, Don Corleone and his friends focus on gambling. Second, social ties among criminals—even among those with different objectives—play an important role in shaping their activities: The actions of Sollozzo, Don Corleone, and his friends in politics are complementary. This is both why Sollozzo offers Don Corleone to join his business, and why Don Corleone is reluctant to accept this offer.

These features are common across a wide variety of social and economic activities. For example, students decide both what and how much to study, and social ties among them play an important role in their success. Similar issues arise when adolescents, farmers and firms decide, respectively, which drugs, fertilizers and technologies to use, and how intensively; or when unions, investigators and facebook users decide, respectively, whether to call a strike, look for evidence, and post in favor or against a certain issue, and how aggressively.

We model these situations as follows[1]: Finitely many players simultaneously decide which action to take. A network $\boldsymbol{G}$ describes the strategic complementarities between players: Each player's best response α_i is the sum of her *autarkic action* β_i (a preference parameter that corresponds to her optimal action absent complementarities) and the linear combination $\theta \sum_j G_{ij}\alpha_j$ of her neighbors' actions, where θ is a measure of the intensity of the strategic complementarities, or *network effects*. In order to isolate the effect of the network topology on welfare—without imposing homogeneous preferences—we take players' autarkic actions to be independently distributed.[2]

[1] Glaeser et al. (1996) were among the first to point out how criminal interconnections act as a social multiplier on aggregate crime. In contrast to their model—which restricts attention to simple network structures—and following the literature that started with Calvó-Armengol and Zenou (2004), we examine a model that admits any social network; see Jackson et al. (2015) for an overview of this literature.

[2] The focus on independently-distributed preferences is common in the literature; see for example Acemoglu et al. (2012) and Acemoglu et al. (2016).

Ballester et al. (2006) showed that this game admits a unique Nash equilibrium, and that—when (i) network effects are not too strong and (ii) autarkic actions are homogeneous—players' equilibrium actions are proportional to their Bonacich centralities (Katz 1953, Bonacich 1987). This result has been widely applied to understand social outcomes and to design polices to improve them. For example, Calvó-Armengol and Jackson (2004) use this result to describe the determinants of different employment dynamics in labor markets, Ballester et al. (2006) to characterize effective criminal deterrence policies, and Candogan et al. (2012) to describe the optimal pricing strategies of a monopolist serving a networked market.

The main contribution of this paper is to characterize geometrically the extent to which a player benefits from her complementarities with others as a function of her position in the network $\boldsymbol{G}$. This is the relevant statistic in many contexts—e.g. a social planner designing a policy to increase welfare, individuals deciding whether to form new partnerships, and a social media platform setting individualized entry fees to maximize profits—and it is not systematically related to the Bonacich centrality measure.

The main result of this paper is that each player's expected utility—before autarkic actions are realized—is proportional to her *cycle centrality*: A new statistic that measures the extent to which other players' actions have *simultaneous* effects on her actions via different network walks. Absent preference correlations—that is, correlations in autarkic actions—these effects are the only source of correlation among players' equilibrium actions, and they determine the extent to which players benefit from their complementarities with others.[3]

As we illustrate, the player who takes *the highest action* when preferences are homogeneous—that is, the most Bonacich-central player—can also be the one who *benefits the least* from her strategic complementarities with others—that is, the least cycle-central player. Hence, while Bonacich centrality has proved extremely useful in understanding the determinants of players' *actions*, our result suggests that it is not the right measure for understanding the connections between social structure and *welfare*, and that cycle centrality can enrich our understanding in this regard.

When strategic complementarities are *pairwise symmetric*,[4] a player's cycle centrality is simply a weighted sum of the number of network cycles that she is in; for arbitrary strategic

[3]Relatedly, Elliott and Golub, (2015, 2017) show how the extent to which a player contributes to the existence of certain network cycles determines how *essential* she is for the scope of cooperation in negotiations over the provision of public goods. Whereas the crucial aspect about the geometry of the network in their case is the strength of arbitrarily long cycles (which determine its spectral radius), the contribution of each cycle to *cycle centrality* is geometrically decreasing in its length.

[4]Strategic complementarities are *pairwise symmetric* if the network that captures them is undirected.

complementarities, it is a weighted sum of the *circles*—that is, the *pairs* of walks from her to another player—that she is in. Whereas the widely-used Bonacich centrality measures how actions propagate through the network,[5] cycle centrality measures how the *variance of actions* propagates through the network: A player's cycle centrality is the sum—across all players j—of the derivative of *the variance of* her equilibrium action with respect to *the variance of* j's autarkic action.

The main insights of this paper are twofold. First, the way in which the variance of equilibrium actions propagates through the network has important welfare implifications, because it determines the extent to which equilibrium actions are correlated across players, and hence the extent to which each player benefits from her complementarities with others. Second, in the standard network game that we consider, the way in which the variance of actions propagates through the network can be characterized geometrically in terms of network cycles. The combination of these two insights provides the connection between network cycles and welfare that we characterize in this paper.

A central objective of the social and economics networks literature is to relate different network statistics to social phenomena (recent examples include Banerjee et al. 2016, Bloch et al. 2016, Ambrus et al. 2017, Baqaee and Farhi 2017, Demange 2017, Elliott and Golub 2017, Galeotti et al. 2017, Golub and Morris 2017 and Leister 2017).[6] To the best of our knowledge, the centrality measure that we identify as being important for welfare—*cycle centrality*—is not systematically related to any of the existing statistics. Hence, our result suggests that cycle centrality has the potential to yield new insights about the effects of social structure on welfare.

The rest of this article is organized as follows. In § 2 we present the model, together with the definitions of *walks*, *circles* and *cycles*. In § 3 we present our main result—each player's expected utility is proportional to her cycle centrality—and we discuss the contrasts between cycle centrality and the widely-used Bonacich centrality. As an application, in § 4 we illustrate how a social planner can use information on players' cycle centralities to design effective policies. Finally, we conclude in § 5.

[5] A player's Bonacich centrality is the sum—across all players j—of the derivative of her equilibrium action with respect to j's autarkic action.

[6] For recent surveys of network-centrality related economic applications, see for example Acemoglu et al. (2016), Golub and Sadler (2016), Zenou (2016) and Jackson et al. (2017).

2 Model

2.1 Preferences

Each player i chooses her action $\alpha_i \in \mathbb{R}$ to maximize her utility

(1) $$u_i = \beta_i \alpha_i - \frac{1}{2}\alpha_i^2 + \theta \alpha_i \sum_j G_{i,j}\alpha_j,$$

where $G_{i,j} \in \mathbb{R}$ is the element in the ith row and jth column of the matrix $\boldsymbol{G}$ that captures the strategic complementarities between players (with $G_{i,i} = 0$ for all players i) and where $\theta > 0$ measures the intensity of the network effects.[7]

Player i's best response function is $\alpha_i = \beta_i + \theta \sum_j G_{i,j}\alpha_j$. Note, in particular, that β_i is i's optimal action absent network effects; for this reason, we refer to β_i as i's *autarkic action*. Players' autarkic actions are identically and independently distributed, with $\mathbb{E}(\beta_i) = 0$ and $\mathbb{E}(\beta_i^2) = \sigma^2$.

Note 2.1. We don't restrict the sign of players' actions. We interpret a positive action as corresponding to pursuing one option (e.g. narcotics) and a negative action as pursuing another option (e.g. gambling). With this interpretation, player i's positive strategic complementarity with player j is interpreted as follows: The more player j pursues gambling, the more it pays for player i to pursue gambling as well (and the less it pays for her to pursue narcotics).

Note 2.2. The assumption that autarkic actions are identically and independently distributed with zero mean is common in the literature; see for example Acemoglu et al. (2016). The assumption that all players' autarkic actions are equally distributed is not crucial for our characterization of expected utility in terms of network cycles (see Note 3.2), but the assumptions that they are independently distributed and that their expected value is zero are. The former assumption allows us to focus on how the network alone generates correlations among players' actions. The latter assumption is natural in the contexts that we consider.

2.2 Nash Equilibrium

We focus on the Nash equilibrium of the one-shot game in which each player chooses $\alpha_i \in \mathbb{R}$ simultaneously. The system of best replies is $(\boldsymbol{I} - \theta \boldsymbol{G})\alpha = \beta$. We require that network effects be sufficiently small; formally, we assume that the spectral radius of $\theta \boldsymbol{G}$ is less than one,

[7] The restriction that θ is positive is without loss of generality, because we do not restrict the signs of the entries of $\boldsymbol{G}$.

so that the *Leontief matrix* $\boldsymbol{M} = (\boldsymbol{I} - \theta\boldsymbol{G})^{-1} = \sum_{k=0}^{\infty} \theta^k \boldsymbol{G}^k$ is well defined, and the unique Nash equilibrium action profile is $\boldsymbol{M}\beta$. This assumption also implies that the equilibrium is *asymptotically stable*; see for example Bramoullé and Kranton (2016).

2.3 Definitions: Walks, Circles and Cycles

In the context of a weighted graph, a *walk* is an ordered set of nodes. A *walk from i to j* is a walk that starts at i and ends at j. The *length of a walk* is the number of links that it involves. The *weight of a walk* is the product of the weights of its links. Figure 1 illustrates a walk of length 3.

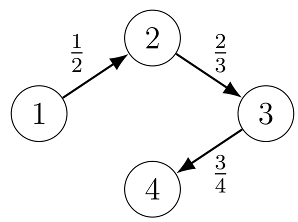


Figure 1: A walk from 1 to 4 of length 3 with weight $\frac{1}{2}\frac{2}{3}\frac{3}{4} = \frac{1}{4}$.

A *k-cycle* is a walk of length k from a node to itself. A *k-circle from i to j* is an ordered set of two walks from i to j whose combined length is k. The *weight of a k-circle* is the product of the weights of its two associated walks. Figure 2 illustrates a 4-circle.

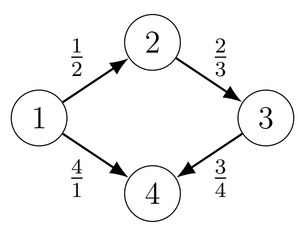


Figure 2: A 4-circle from 1 to 4 with weight $\frac{1}{4}4 = 1$.

We denote by D_i^k the sum of the weights of all different k-circles that start at i. In particular, in the pairwise-symmetric case (i.e., when $G_{i,j} = G_{j,i}$) $D_i^k = (k+1)C_i^k$, where C_i^k is the sum of the weights of all of k-cycles that start and end at i.[8]

[8]To see this note that, for any given k-cycle $C := \{i_1, i_2, \ldots, i_k, i_1\}$, there are $k+1$ associated k-circles that start at i_1: $(\{i_1, i_2, \ldots, i_1\}, \{i_1\}), (\{i_1, i_2, \ldots, i_k\}, \{i_1, i_k\}), \ldots, (\{i_1\}, \{i_1, i_k, i_{k-1}, \ldots, i_1\})$. And the weight of each of these k-circles is the same as the weight of the k-cycle C.

Definition 2.1 defines a new network centrality measure that—as Theorem 3.1 shows—determines the value of each network position.

Definition 2.1. Player i's *cycle centrality* is $c_i := \sum_{k=0}^{\infty} \theta^k D_i^k$.

3 Cycle Centrality Determines Welfare

In § 3.1 we present the main result of this paper: The extent to which a player benefits from her strategic complementarities with others is proportional to her cycle centrality. In § 3.2 we illustrate—via a simple example—the contrasts between cycle centrality and the widely-used Bonacich centrality.

3.1 Main Result

Theorem 3.1. *Each player's expected utility is proportional to her cycle centrality.*

Proof. Player i's first order condition is

$$u_i = \frac{1}{2}\alpha_i^2 = \frac{1}{2}(\boldsymbol{M}\beta)_i^2 = \frac{1}{2}\sum_{j,k} M_{ij}\beta_j M_{ik}\beta_k. \tag{2}$$

Taking expectations, since the β_i's are iid with $\mathbb{E}(\beta_i) = 0$ and $\mathbb{E}(\beta_i^2) = \sigma^2$, we get

$$\mathbb{E}(u_i) = \frac{\sigma^2}{2}\sum_j M_{ij}^2. \tag{3}$$

Recalling that $M_{ij} = \sum_{\ell=0}^{\infty} \theta^\ell (\boldsymbol{G}^\ell)_{ij}$, noting that $(\boldsymbol{G}^k)_{ij}$ is the sum of the weights of all k-walks from i to j, and collecting terms we get

$$M_{ij}^2 = \sum_{\ell=0}^{\infty}\sum_{m=0}^{\infty} \theta^{\ell+m}(\boldsymbol{G}^\ell)_{ij}(\boldsymbol{G}^m)_{ij} = \sum_{k=0}^{\infty}\sum_{\ell=0}^{k} \theta^k (\boldsymbol{G}^\ell)_{ij}(\boldsymbol{G}^{k-\ell})_{ij}. \tag{4}$$

Summing over j and noting that $D_i^k = \sum_j \sum_{\ell=0}^{k} (\boldsymbol{G}^\ell)_{ij}(\boldsymbol{G}^{k-\ell})_{ij}$ gives the result. □

Note 3.1. Player i's Bonacich centrality is $b_i := \sum_{k=0}^{\infty} \theta^k W_i^k$, where W_i^k denotes the sum of the weights of all different k-walks that start at i. Equivalently, player i's Bonacich centrality is (i) the sum of the elements in the ith row of the Leontief matrix $\boldsymbol{M}$, and (ii) the sum—across all players j—of the derivative of her equilibrium action with respect to j's autarkic action.

In contrast, player i's cycle centrality is (i) the sum of *the squares* of the entries of the ith row of this matrix, and (ii) the sum—across all players j—of the derivative of *the variance of* her equilibrium action with respect to *the variance of* j's autarkic action. In § 3.2, we further illustrate the contrasts between these two centrality measures with a simple example.

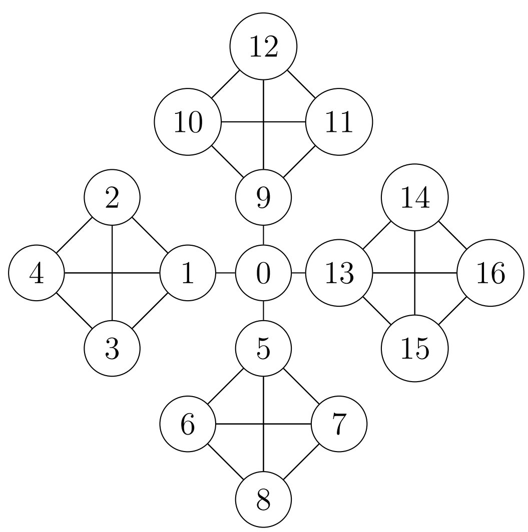
Note 3.2. If players' autarkic actions are not identically distributed, player i's expected utility is proportional to $\sum_{k=0}^{\infty} \theta^k \sum_j D_{ij}^k \sigma_j^2$, where D_{ij}^k denotes the sum of the weights of all k-circles from i to j, and $\sigma_j^2 := \mathbb{E}(\beta_j^2)$. Intuitively, the weight of k-circles from i to j is proportional to the variance of j's autarkic action.

3.2 Cycle Centrality not Systematically Related to Bonacich Centrality

As discussed in Note 3.1, cycle centrality and Bonacich centrality are not systematically related. In this section we illustrate this point via a highly symmetric network in which the most Bonacich-central player can be the least cycle central.

Consider the network depicted in Figure 3. Note that players $1, 5, 9$ and 13 play the same role in this network.[9] Similarly, players $2, 3, 4, 6, 7, 8, 10, 11, 12, 14, 15$ and 16 play the same role, and there is no player that plays the same role as player 0. Note that players who play the same role have the same centrality.

k	C_0^k	C_1^k	C_2^k
1	0	0	0
2	**4**	**4**	3
3	0	**6**	**6**
4	28	**31**	22
5	24	**72**	64



k	W_0^k	W_1^k	W_2^k
1	**4**	**4**	3
2	**16**	13	10
3	**56**	42	33
4	**184**	151	112
5	**604**	520	375

Figure 3: A network with three player roles, together with a sample of the number of k-cycles (table on the left) and k-walks (table on the right) emanating from each player role. The maximum (minimum) in each row is in bold (gray).

The left and right tables in Figure 3 indicate, respectively, the number of k-cycles and k-walks emanating from each of these roles (for the first five values of k, and where we

[9]Formally, two nodes i and j of a network g play the same role if there is a *symmetry of* g—that is, an automorphism of g, or a renaming of the players that preserves edges—that maps i to j.

identify players $0, 1$ and 2 with each of the possible three player roles in this network). Note that, for all values of k depicted, more walks emanate from player 0 than from any other player. Indeed, this is true for every $k > 0$, so for every intensity θ of network effects, player 0 is the most Bonacich-central player.

In contrast, for all values of k depicted, more k-cycles emanate from player 1 than from any other player. As a consequence, for all small enough intensities θ of network effects, player 1 is the most cycle-central player. For large enough values of k, however (e.g. $k \geq 15$), more k-cycles emanate from player 0 than from any other player, and hence for all high enough intensities θ of network effects, player 0 is the most cycle-central player.

The situation of player 2 is a bit more subtle: On the one hand, for all values of k, less k-cycles emanate from player 2 than from player 1, so player 2 is less cycle central than player 1 for all intensities of network effects. On the other hand, more k-cycles emanate from player 2 than from player 0 for some values of k (e.g. $k = 3, 5$) but less k-cycles emanate from player 2 than from player 0 for other values of k (e.g. $k = 2, 4$). As a consequence, player 2 is more cycle central than player 0 for some network-effect intensities (e.g. $\theta = .2$) but less cycle central than player 0 for other network-effect intensities (e.g. $\theta = .25$).

Table 1 illustrates these phenomena by depicting both the cycle and the Bonacich centralities for each of the three player types and a sample of network effects θ (below the maximum value $\theta \approx .3$ of network effects for which these measures are well defined). Note in particular how, for intermediate values of network effects (e.g. $\theta = .2$), player 0 is the least cycle-central player while being the most Bonacich central.

	$\theta = .1$		$\theta = .2$		$\theta = .25$		$\theta = .28$	
Player Type	c_i	b_i	c_i	b_i	c_i	b_i		
0	1.14	**1.64**	2.08	**3.75**	5.89	**8.33**	**39.52**	**24.52**
1	**1.17**	1.60	**2.47**	3.44	**6.91**	7.33	35.08	21.00
2	1.13	1.45	2.16	2.81	5.36	5.67	22.33	15.64

Table 1: Cycle and Bonacich centralities for four different intensities θ of network effects. The maximum (minimum) value of each column is in bold (gray).

Intuitively, cycle centrality is sensitive to the fact that more intermediate-length cycles emanate from players 1 and 2 than from player 0, even if more walks (of all lengths) emanate from player 0 than from players 1 and 2. For this reason, when intermediate-length cycles have relatively high weight in the cycle centrality measure—that is, when network effects are of intermediate intensity—player 0 is the least cycle central, even though she is the most

Bonacich central.

4 Social Planner Targets the Most Cycle-Central Player

In this section we illustrate how a planner can use the cycle centrality network statistic to maximize the effectiveness of her policies. This analysis is motivated by Ballester et al. (2004), who extensively analyze how a social planner willing to reduce criminal activity can use Bonacich centrality to increase the effectiveness of her policies. For simplicity, in this section we focus on the case in which strategic complementarities are pairwise symmetric (we discuss the case in which complementarities are not pairwise symmetric in Note 4.2).[10]

Following the celebrated criminal-market application of Ballester et al. (2004), we focus on the problem of a planner who wants to minimize the expected sum of players' utilities; this could be, for example, because she wants to minimize the number of active criminals, and potential criminals choose to become active if their expected utility of doing so is higher than their outside option.[11] An analogous analysis goes through if the planner wants to *maximize* the expected sum of players' utilities; this could be, for example, simply because the planner is benevolent, or because she wants to maximize the profits of a social media platform who is able to charge individualized entry fees.

In order to achieve her objective, the social planner can—before preferences are realized—commit to disincentivizing the action of one player, which reduces her autarkic action's variance by $\epsilon > 0$ (where ϵ is smaller than the variance of each player's autarkic action). This could represent, for example, the planner's ability to focus her investigative efforts towards one criminal.[12] We discuss alternative policy options in Note 4.3 and Note 4.4.

Who should the social planner target? The variance of the equilibrium action of the targeted player will be reduced, which—in turn—will reduce the variance of the actions of others. Hence, since each player's expected utility is the variance of her equilibrium action,

[10]For recent related exercises, see for example Galeotti et al. (2017), König et al. (2017) and Leister (2017).

[11]See e.g Ballester et al. (2004) and Calvó-Armengol and Jackson (2004) for formal models along these lines. Whereas in these models potential criminals decide to enter once they know both their preferences and their position in the network, the simplest version of our story requires that potential criminals choose whether to become active before they know both their preferences and which network position they will occupy. In this case, all potential criminals are homogeneous at the time when they choose whether or not to become active, and it is an equilibrium for all of them to become active if the expected utility of doing so is bigger than their outside option.

[12]See Zenou (2016) for a survey discussing several implementations of similiar exercises.

the optimal target is the player for whom this cumulative effect is highest. As shown by Proposition 4.1, this is the player with the highest cycle centrality.

Proposition 4.1. *The optimal target is the player with the highest cycle centrality.*

Proof. Letting $\mathbb{E}(\beta_i^2) = \sigma_i^2$, without assuming that $\sigma_i^2 = \sigma^2$ for all i, Equation 3 reads

$$\mathbb{E}(u_i) = \frac{1}{2}\sum_j M_{ij}^2\sigma_j^2 = \frac{1}{2}\sum_{k=0}^{\infty}\theta^k\sum_j\sigma_j^2\sum_{\ell=0}^{k}(\boldsymbol{G}^{\ell})_{ij}(\boldsymbol{G}^{k-\ell})_{ij}.$$

where we have used Equation 4. Noting that $D_{ij}^k := \sum_{\ell=0}^{k}(\boldsymbol{G}^{\ell})_{ij}(\boldsymbol{G}^{k-\ell})_{ij}$ is the sum of the weights of all k-circles from i to j, we obtain that

$$\mathbb{E}(u_i) = \frac{1}{2}\sum_{k=0}^{\infty}\theta^k\sum_j D_{ij}^k\sigma_j^2 \tag{5}$$

Summing over all i yields

$$\mathbb{E}\left(\sum_i u_i\right) = \frac{1}{2}\sum_{k=0}^{\infty}\theta^k\sum_{i,j}D_{ij}^k\sigma_j^2. \tag{6}$$

When $\boldsymbol{G}$ is symmetric, we have that $\sum_i D_{ij}^k = \sum_i D_{ji}^k = D_j^k$, and hence we can write Equation 5 as

$$\mathbb{E}\left(\sum_i u_i\right) = \frac{1}{2}\sum_j\sigma_j^2 c_j. \tag{7}$$

In particular, the derivative of $\mathbb{E}\left(\sum_i u_i\right)$ with respect to σ_j^2 is j's cycle centrality. □

Note 4.1. Proposition 4.1 holds irrespectively of whether individual's autarkic actions are identically distributed. Intuitively, the expected sum of players' utilities is linear in the variances of the autarkic actions, so the effect of a marginal increase in the variance of the autarkic action of one player on the expected sum of utilities is independent of the initial variances.

Note 4.2. When strategic complementarities are not pairwise symmetric, it follows from Equation 6 that the planner's optimal target is the player j for whom $\sum_{k=0}^{\infty}\theta^k\sum_i D_{ij}^k$ is highest. This is the player who is most cycle central in the transpose of $\boldsymbol{G}$.

Note 4.3. A social planner who—instead of being able to incentivize one individual only—can choose to reduce the variance of each player i by x_i subject to the budget constraint $\sum_{i\in N}x_i^2 = 1$, spends on player i a fraction of her budget that is proportional to i's cycle centrality.

Note 4.4. An alternative way to illustrate how measuring cycle centrality can be useful to design efficient policies is to focus—as in Ballester et al. (2006), for example—on the case in which the social planner can remove one player from the network altogether in order to minimize the expected sum of player's utilities.[13] By Equation 7, the *key player*—that is, the optimal target of the social planner—is the one whose removal decreases the weighted sum $\sum_j \sigma_j^2 c_j$ of players' cycle centralities the most.

As in the key-player policy of Ballester et al. (2006), when one player is removed from the network, the impact on this sum is twofold: First, the expected sum of utilities decreases by the expected utility of the player removed. Second, the cycle centrality of all other players changes. Because of this second—indirect—effect, the key player need not be the player with the highest cycle centrality.

5 Conclusion

In many economic and social settings, players' actions are complementary and understanding the determinants their welfare important. The main result of this paper is that, in a setting in which (i) players' preferences are independently distributed, and (ii) a network specifies the existing strategic complementarities, the extent to which a player is present in cycles of this network—her *cycle centrality*—determines the extent to which she benefits from her strategic complementarities with others. This paper also illustrates how a social planner can use data on cycle centrality to design effective policies to maximize or minimize utilitarian welfare.

Since existing centrality measures are not systematically related to cycle centrality, the results of this paper suggest that this network statistic has the potential for being an important new tool for understanding the connections between social structure and social and economic outcomes. The tight connection between cycle centrality and welfare illustrated in this article relies on a widely-used—but special—linear-quadratic model. Elliott and Golub (2017) show how many of the insights developed by the literature that uses this model apply much more generally. We leave the exploration of the connection between cycle centrality and welfare in more general settings for future research.

[13] Ballester et al. (2006) characterize the optimal policy of a planner who wants to minimize the sum of all *actions*.

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