

MA144A, HOMEWORK 3
DUE THURSDAY, OCTOBER 25TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

- (1) Prove the Dominated Convergence Theorem using the Monotone Convergence Theorem.
- (2) Find a sequence of independent random variables (X_1, X_2, \dots) with $\mathbb{P}[X_n \in \{-n, n, 0\}] = 1$, $\mathbb{E}[X_n] = 0$, and such that the weak LLN holds but not the strong: for $Y_n = \frac{1}{n} \sum_{k \leq n} X_k$ it holds that $\mathbb{P}[|Y_n| \geq \varepsilon] \rightarrow 0$ but $\mathbb{P}[\lim_n Y_n = 0] \neq 1$.
- (3) Prove that there exists a *simply normal number*: a real number $x \in [0, 1]$ such that for any $d > 1$ and $a \in \{0, \dots, d-1\}$ the digit a occurs in the base d representation of x with asymptotic frequency $1/d$:

$$\lim_n \frac{\text{number of times } a \text{ occurs in the first } n \text{ digits of } x}{n} = \frac{1}{d}.$$