

MA144A, HOMEWORK 7
DUE TUESDAY, DECEMBER 4TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Let \mathcal{T}_3 be the set of nodes of the undirected 3-regular tree. This is the unique undirected graph with no cycles and in which each node has degree 3. The simple random walk on \mathcal{T}_3 is the Markov chain that starts from some $s_0 \in \mathcal{T}_3$ and, at each step, transitions to each of the three neighboring nodes with probability $1/3$.

- (1) Prove that the simple random walk on \mathcal{T}_3 is transient.
- (2) Prove that the simple random walk on \mathcal{T}_3 has a non-trivial shift-invariant σ -algebra (that is, that there is some shift-invariant event with probability strictly between 0 and 1).