PS/Ec 172, Set 4
Due Friday, May 7th

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Chicken, mixed and correlated equilibria.* Consider the game of Chicken, as parametrized by $a, b, c > 0$ and $b > a$:

<table>
<thead>
<tr>
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<th>$Y$</th>
<th>$D$</th>
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<tbody>
<tr>
<td>$Y$</td>
<td>$a$, $a$</td>
<td>$0$, $b$</td>
</tr>
<tr>
<td>$D$</td>
<td>$b$, $0$</td>
<td>$-c$, $-c$</td>
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(a) 20 points. Explain why every finite game has a correlated equilibrium.

(b) 20 points. Find all completely mixed Nash equilibria.

(c) 20 points. For which values of $a, b, c$ does there exist a symmetric correlated equilibrium with total expected utility larger than that of any pure equilibrium? In this game, a correlated equilibrium is symmetric if the probabilities of $(Y, D)$ and $(D, Y)$ are the same.

(2) 40 points. Construct an example of a knowledge space with two players, a finite set of states of the world, an event $A$ and a state of the world $\omega$ such that $\omega \in K_1 A$, $\omega \in K_2 A$, $\omega \in K_1 K_2 A$, $\omega \in K_2 K_1 A$, but $\omega \notin K_1 K_2 K_1 A$.

(3) *Bonus question.* A prisoner escapes to $\mathbb{Z}^2$ on Sunday. Every day he must move either one up (i.e., add $(0, 1)$ to his location) or one to the right (add $(1, 0)$), except on Saturdays, when he must rest. The detective can, once a day, check one element of $\mathbb{Z}^2$ and see if the prisoner is there. If she finds him then she wins. He wins if she never finds him.

Formally, the prisoner’s strategy is an element $(z, f) \in \mathbb{Z}^2 \times \{(1, 0), (0, 1), (0, 0)\}^\mathbb{N}$ such that $f(n) = (0, 0)$ whenever $n \equiv 0 \mod 7$, and $f(n) \in \{(1, 0), (0, 1)\}$ otherwise. The detective’s strategy is a sequence $\{z_n\}_{n \in \mathbb{N}}$ with $z_n \in \mathbb{Z}^2$.

The prisoner’s current location when using strategy $(z, f)$ is

$$\ell_n = z + \sum_{k=1}^{n-1} f(k).$$

The detective wins if $\ell_n = z_n$ for some $n$. The prisoner wins otherwise.

(a) 1 point. Show that the detective has a winning strategy.

(b) 1 point. Show that if we remove the requirement that the prisoner rests on Saturdays then the detective does not have a winning strategy.

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