Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) Bundling. Anya walks into a store with the intention of buying a loaf of bread and a stick of butter. Her valuations for the two items are chosen independently from the uniform distribution on $[0, 1]$. Sharne, the store owner, has to set the prices. We assume that Anya will buy for any price that is lower than her valuation.

(a) 20 points. Assume first that Sharne sets a price $b_l$ for the loaf and $b_s$ for the stick. What is her expected revenue, as a function of $b_l$ and $b_s$?

(b) 5 points. What is the maximal expected revenue she can get?

(c) 20 points. Sharne now decides to bundle: she sets a price $b_b$ for buying both items together, and does not offer each one of them separately. That is, she offers Anya to either buy both for $b_b$, or else get neither. What is her expected revenue, as a function of $b_b$?

(d) 5 points. What is the maximal expected revenue she can get now?

(2) Repeated prisoner’s dilemma. Let $G_0$ be the following version of prisoner’s dilemma:

\[
\begin{array}{cc}
  & D & C \\
  D & 0, 0 & 3, -4 \\
  C & -4, 3 & 2, 2 \\
\end{array}
\]

Let $G$ be the repeated game in which $G_0$ is played for $T$ periods. The strategy tit-for-tat is the strategy in which a player plays $C$ in the first period, and henceforth always plays the same strategy that the other player played in the previous round. Let $s$ be the strategy profile in which both players play tit-for-tat.

(a) 15 points. Let $T = 10$, and let the players’ utilities be the sum of their stage utilities. Is $s$ an equilibrium?

(b) 20 points. Let $T = \infty$, and let the players’ utilities be $\delta$-discounting. For which values of $\delta$ is $s$ an equilibrium?

(c) 15 points. Again, let $T = \infty$, and let the players’ utilities be $\delta$-discounting. Consider now the strategy profile $s'$ in which each player plays $C$ as long as no $D$ has yet been played by any player, and otherwise plays $D$. For which values of $\delta$ is $s'$ an equilibrium?
(3) **Bonus question: Mind reading (with high probability).** Ali and Fatima play a game. Ali picks a finite subset $F \subseteq \mathbb{N}$, and Fatima picks an $n \in \mathbb{N}$. Ali wins if $n \in F$, and Fatima wins otherwise.

Before choosing her $n$, Fatima picks any subset $S \subseteq \mathbb{N}$. For example, $S$ could be the even numbers. Ali reveals to Fatima the intersection $S \cap F$; we assume he does so truthfully. Fatima can now choose her number $n$. It can depend on Ali’s answer, but it cannot be in $S$. She wins if $n \not\in F$, and otherwise Ali wins.

Formally, a pure strategy for Ali is a choice of $F$. A pure strategy for Fatima is a choice of $S$, plus a function from subsets of $S$ to $\mathbb{N} \setminus S$; this is the function that specifies $n$ given $S \cap F$.

(a) 1 point. Show that for every pure strategy of Fatima there is a pure strategy of Ali that ensures that he wins, and that for every pure strategy of Ali there is a pure strategy of Fatima that ensures that she wins.

(b) 1 point. Show that Fatima has a mixed strategy (i.e., a randomly picked strategy) such that for every $F$, her probability of winning is at least $\frac{2020}{2021}$. 