PS/Ec 172, Homework 6
Due Thursday, November 29th

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Bundling.* George walks into a store with the intention of buying a loaf of bread and a stick of butter. His valuations for the two items are chosen independently from the uniform distribution on $[0, 1]$. Kavya, the store owner, has to set the prices. We assume that George will buy for any price that is lower than his valuation.

   (a) *10 points.* Assume first that Kavya sets a price b_l for the loaf and b_s for the stick. What is her expected revenue, as a function of b_l and b_s?

   (b) *5 points.* What is the maximal expected revenue she can get?

   (c) *20 points.* Kavya now decides to *bundle*: she sets a price b_b for buying both items together, and does not offer each one of them separately. That is, she offers George to either buy both for b_b, or else get neither. What is her expected revenue, as a function of b_b?

   (d) *5 points.* What is the maximal expected revenue she can get now?

   (e) *Bonus question (1 point).* Assume now that Kavya sets three different prices: b_l for the loaf, b_s for the stick, and b_b for both, so that George can choose if to buy just the loaf (for b_l), just the stick (for b_s), or both (for b_b). Assume that he will choose to buy whichever items maximize his utility, which is his value for the bought items minus the price paid. What is the maximal expected revenue she can get now?

(2) *A market for lemons.* Alex is shopping for a used car in Bob's used car lot. Every car that arrives at the lot is with probability one half in bad condition (worth \$1,000), and with probability one half in good condition (worth \$3,000). Bob observes the condition of the car, but Alex does not. Bob sets a price for the car (an integral number of dollars between \$0 and \$10,000), and Alex learns this price and decides whether or not to buy.

   If Alex decides to buy, his utility is the value of the car minus the price. Otherwise his utility is zero. Bob's utility is explained below.

   (a) *20 points.* Draw the tree of this extensive form game. That is, draw the graph whose vertices are the histories and whose edges correspond to possible actions. What are Alex's information sets?

   (b) *20 points.* Assume first that Bob's utility, if Alex buys, is the price, minus the value of the car, plus \$100. If no trade occurs her utility is zero (this describes a situation in which Bob buys the cars for a \$100 discount).

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Construct a pure equilibrium in which good cars are sold, or explain why no such equilibria exist.

(c) *20 points.* Assume now that Bob's utility is \$1 for a sale at or above the car's worth, -\$1 for a sale below the car's worth, and \$0 if there is no sale (this describes a situation in which Bob earns a fixed commission per sale).
Construct a pure equilibrium in which good cars are sold, or explain why no such equilibria exist.