PS/EC 172, SET 5
DUE FRIDAY, MAY 19TH AT 11:59PM
RESUBMISSION DUE FRIDAY, JUNE 7TH AT 11:59PM

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Reserve prices*. Michael and Thierno would both like to buy an item owned by Nishka. Michael and Thierno's valuations are chosen independently from the uniform distribution on $[0,1]$, and each is known only to himself.
   (a) *20 points.* What is Nishka's expected revenue from a second price auction?

   (b) *20 points.* Nishka now introduces a *reserve price* $b_r \in [0,1]$: if the maximum bid is under b_r then the auction is canceled, no one gets the item and no one pays. Otherwise, the winner pays the maximum of b_r and the loser's bid. What is her expected revenue, as a function of b_r?

   (c) *10 points.* What is the maximal expected revenue she can get by choosing b_r optimally?

(2) *Bundling*. Moya walks into a store with the intention of buying a loaf of bread and a stick of butter. Her valuations for the two items are chosen independently from the uniform distribution on $[0,1]$. Lilly, the store owner, has to set the prices. We assume that Moya will buy for any price that is lower than her valuation.
   (a) *20 points.* Assume first that Lilly sets a price b_l for the loaf and b_s for the stick. What is her expected revenue, as a function of b_l and b_s?

   (b) *5 points.* What is the maximal expected revenue she can get?

   (c) *20 points.* Lilly now decides to *bundle*: she sets a price b_b for buying both items together, and does not offer each one of them separately. That is, she offers Moya to either buy both for b_b, or else get neither. What is her expected revenue, as a function of b_b?

   (d) *5 points.* What is the maximal expected revenue she can get now?

(3) *Bonus: a riddle with both prisoners and hats (Gabay-O'Connor game).* There are n prisoners standing in a line. The first can observe all the rest. The second can observe all except the first, etc. Each is given either a red or a blue hat which he cannot see. Now, starting with the first prisoner, each in turn has to guess the color of his hat, a guess which the rest can hear.
   (a) *1 point.* The prisoners are allowed to decide on a strategy ahead of time. Find one in which they all guess the color correctly, except maybe the first prisoner.

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(b) *1 point.* Do the same, but for an infinite line of prisoners.

(c) *1 point.* For an infinite line of deaf prisoners, find a strategy in which at most finitely many of them guess incorrectly.