PS/Ec 172
Final Exam

Please take three hours to complete this exam. Collaboration is not allowed, except on the bonus question, which you can discuss with others, given that this discussion takes place in public conversations on Campuswire. Also, there is no time limit for completing the bonus question, and you can read it (on the last page of the exam) before starting the three hour clock.

You may consult the lecture notes, your own notes or any textbook, but not solutions to previous exams. There is no need to prove any statements that you make, unless this is explicitly requested (e.g., "explain..."), in which case you must provide an answer that is no longer than one paragraph. The exam is due at 1pm on Thursday, December 13$^{\text{th}}$, and is to be handed in at the instructor's office, Baxter 213, from 10am that day.

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(1) The Chandler Cafeteria has started offering *escargot*. Tzarina and Gemma are eager to try it, but both are afraid that it is awful. A-priori, there is a 10% chance that it is awful (A) and a 90% chance that it is good (G).

There are time periods $t \in \{0, 1, 2, \ldots\}$, and in each time period they each have to simultaneously decide whether to eat (E) it or not (N). Once one of them has decided to eat it, the quality of the escargot is revealed to both and never changes.

The stage utility (at any period t) for taking action a with escargot of quality q is

$$u_t(a, q) = \begin{cases} 0 & \text{if } a = N \\ 1 & \text{if } a = E \text{ and } q = G \\ -40 & \text{if } a = E \text{ and } q = A \end{cases}.$$

A player's total utility in the game is

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t u_t$$

for $\delta = 9/10$.

The following questions are for 10 points each.

(a) Explain why there are no equilibria in which neither of the players ever eat.

(b) Find a pure Nash equilibrium in which Tzarina's expected utility is higher than Gemma's.

(c) Find a symmetric mixed Nash equilibrium.

(d) Find a symmetric correlated equilibrium in which the players' expected utilities are higher than in the symmetric mixed Nash equilibrium.

(2) Consider the following extensive form game with incomplete information, played by a student, a teacher and an employer.
- The student decides whether or not to study.
- Studying costs 1/3, while not studying costs nothing.
- If she studies then she will be able to solve the exam.
- If she does not study then she will be able to solve the exam with probability $0 < p < 1$.
- She takes the exam, and solves it if she can.
- The teacher decides whether to read the exam.
- Reading the exam costs 1/4.
- If the teacher reads the exam he knows whether it was solved.
- The teacher has to give a grade: either pass or fail.
- The teacher gets utility 1 from passing a solved exam or failing an exam that was not solved. Otherwise he gets utility 0.
- The employer observes the grade, and gets utility 1 from hiring a student who can solve the exam, utility -1 from hiring a student who cannot, and utility of 0 from not hiring.
- The student gets utility 1 from getting hired, and utility 0 from not getting hired.

The following questions are for 20 points each.

(a) **Harvard.** For which values of p does there exist an equilibrium in which the student **does not** study, the teacher **does not** read the exam, the teacher **passes** the student and the employer **hires** the student?

(b) **Berkeley.** For which values of p does there exist an equilibrium in which the student **does not** study, the teacher **does** read the exam, the teacher **either passes or fails** the student (depending on how she did on the exam), and the employer **hires** the student only if she passed?

(c) **Caltech.** For which values of p does there exist an equilibrium in which the student **does** study, the teacher **does not** read the exam, the teacher **passes** the student and the employer **hires** the student?

(3) *Bonus question. You may discuss this question in public Campuswire conversations.*

Choose an integer $x \in \{1, 2, \ldots, 10\}$. Denote by $\bar{x}$ the average choice for x made by all the students taking this exam.

As in the midterm, if $\bar{x} = 1$ you will get 5.5 bonus points. You will get x points if $\bar{x} < 4$. Otherwise you will get nothing.