PS/Ec 172
Final Exam

Please take three hours to complete this exam. Collaboration is not allowed, except on the bonus question, which you can discuss with others, given that this discussion takes place in public conversations on Piazza. Also, there is no time limit for completing the bonus question, and you can read it (on the last page of the exam) before starting the three hour clock.

You may consult the lecture notes, your own notes or any textbook, but not solutions to previous exams. There is no need to prove any statements that you make, unless this is explicitly requested (e.g., “explain...”), in which case you must provide an answer that is no longer than one paragraph. The exam is due at 1pm on June 4th.

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(1) Consider the following extensive form game with incomplete information, played by a student, a teacher and an employer.

- The student decides whether or not to study.
- Studying costs \(\frac{1}{3}\), while not studying costs nothing.
- If she studies then she will be able to solve the exam.
- If she does not study then she will be able to solve the exam with probability \(0 < p < 1\).
- She takes the exam, and solves it if she can.
- The teacher decides whether to read the exam.
- The teacher does not know if the student studied.
- Reading the exam costs \(\frac{1}{4}\).
- If the teacher reads the exam he knows whether it was solved.
- The teacher has to give a grade: either pass or fail.
- The teacher gets utility 1 from passing a solved exam or failing an exam that was not solved. Otherwise he gets utility 0.
- The employer observes the grade, and gets utility 1 from hiring a student who can solve the exam, utility -1 from hiring a student who cannot, and utility of 0 from not hiring.
- The student gets utility 1 from getting hired, and utility 0 from not getting hired.

In summary: the student has to decide whether or not to study. The teacher has to decide whether or not to read the exam, and then has to grade it; she does not know if the student studied, and needs to read the exam to know if it was solved. The employer, who observes the grade, has to decide whether or not to hire. The questions below are regarding pure strategies.

The following questions are for 20 points each.

(a) **Harvard.** For which values of \(p\) does there exist an equilibrium in which the student does not study, the teacher does not read the exam, the teacher passes the student and the employer hires the student?

(b) **Berkeley.** For which values of \(p\) does there exist an equilibrium in which the student does not study, the teacher does read the exam, the teacher either passes or fails the student (depending on how she did on the exam), and the employer hires the student if and only if she passed?

(c) **Caltech.** For which values of \(p\) does there exist an equilibrium in which the student does study, the teacher does not read the exam, the teacher passes the student and the employer hires the student?
(2) Consider the following game played by \( n \geq 3 \) players who are sitting in a circle. Each player chooses one of two actions: \( X \) or \( Y \). The players make this choice simultaneously. The payoff to a player is 0 if she chooses the same action as the person on her right, and 1 otherwise.

(a) 5 points. Let \( n \) be even. Find a pure Nash equilibrium or explain why none exist.

(b) 5 points. Let \( n \) be odd. Find a pure Nash equilibrium or explain why none exist.

(c) 10 points. Find a completely mixed Nash equilibrium for those values of \( n \) for which no pure one exists. What is the expected utility to each player?

(d) 10 points. For those values of \( n \) for which no pure Nash equilibria exist, find a correlated equilibrium in which the expected utility to each player is \( 1 - 1/n \).
   Hint: Imagine a benevolent social planner who (privately) chooses a random number (from some distribution), and according to the outcome tells each player (in private) which action to choose; each player learns only the action that she is to play. This is done in such a way that each player has no incentive to disobey, assuming the rest are also obedient.

(3) The Chandler Cafeteria has started offering escargot. Jane and Jason are eager to try it, but both are afraid that it is awful. A-priori, there is a 10% chance that it is awful (\( A \)) and a 90% chance that it is good (\( G \)).

There are time periods \( t \in \{1, 2, \ldots\} \), and in each time period they each have to simultaneously decide whether to eat (\( E \)) it or not (\( N \)). Once one of them has decided to eat it, the quality of the escargot is revealed to both and never changes.

The stage utility (at any period \( t \)) for taking action \( a \) with escargot of quality \( q \) is

\[
u_t(a, q) = \begin{cases} 
0 & \text{if } a = N \\
1 & \text{if } a = E \text{ and } q = G \\
-40 & \text{if } a = E \text{ and } q = A 
\end{cases}
\]

A player’s total utility in the game is

\[
(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_t
\]

for \( \delta = 9/10 \).

(a) 1 point. Consider the strategy profile in which both players always choose not to eat. Explain: is this an equilibrium?

(b) 1 point. Consider the strategy profile in which both players eat in the first period, and then later either eat in all remaining periods or do not eat in all remaining periods, depending on whether the quality is \( G \) or \( A \). Explain: is this an equilibrium?
(c) 8 points. Find a mixed Nash equilibrium in which each player eats with probability $p$ in every period, as long as the quality has not been revealed, and once the quality has been revealed either eats in all remaining periods or does not eat in all remaining periods, depending on whether the quality is $G$ or $A$. 
(4) **Bonus question.** You may discuss this question in public Piazza conversations.

Choose an integer $x \in \{0, 1, 2, \ldots, 10\}$. Denote by $a$ the average choice for $x$ made by all the students taking this exam. As in the midterm, you will get $x$ bonus points if $a < 4$. Otherwise you will get nothing.

Note: bonus points will be added *after* any curving.