PS/Ec 172
Midterm Exam

Please take a contiguous three hour block to complete this exam; you can stop the clock if you have a question regarding that exam that you need answered. Collaboration is not allowed. You may consult the lecture notes, your own notes or any textbook, but not solutions to previous problem sets or exams. There is no need to prove any statements that you make. The exam is due at 1pm pacific time on Friday, April 30^{th}.

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A class of n students are taking an exam. In one of the questions the students are instructed to write an integer between 0 and 10. That is, each student i writes a number $x_i \in \{0, 1, 2, \ldots, 10\}$. The numbers $\{x_i\}_{i=1}^n$ that the students wrote are collected, and the average $a = \frac{1}{n}\sum_i x_i$ is calculated. If $a < 4$, then each student i gets x_i points for that question. Otherwise all the students get no points.

(1) Assume that $n = 100$, and that all students submit their exams simultaneously and without observing the others' answers. We say that a strategy profile is symmetric if all students play the same strategy. That is, $(x_1, \ldots, x_n)$ is symmetric if $x_i = x_j$ for all i, j.
   (a) *10 points.* Which (if any) of the 11 symmetric strategy profiles are equilibria?

   (b) *10 points.* Find a (non-symmetric) pure equilibrium in which some of the students choose 3 and some choose 4.

   (c) *10 points.* Which (if any) of the strategies $\{0, \ldots, 10\}$ is weakly dominated? Which (if any) is strictly dominated?

(2) *20 points.* Assume now that $n = 2$, and that, as before, both students submit their exams simultaneously and without observing the others' answers. Consider the mixed strategy profile in which both student 1 and student 2 choose 3 with probability p and choose 4 with probability $1 - p$. For which p is this a mixed Nash equilibrium?

(3) *20 points.* Assume again that $n = 2$, but that now student 1 submits the exam before student 2, and that student 2 knows what student 1 submitted before making her choice; this is thus an extensive form game with perfect information in which student 1 plays before student 2. Find all pure Nash equilibria of this game, and state which ones are subgame perfect.

(4) Assume again that $n = 2$. The students complete the exam without observing each other's choices. When they get their exams back they learn whether they got points for that question.

   Formally, the students' knowledge space after getting their exam back is $\Omega = \{0, \ldots, 10\}^2$. At $(x_1, x_2) \in \Omega$ student 1 knows x_1, and student 2 knows x_2, and additionally both know whether or not $(x_1 + x_2)/2 < 4$.

   We will say that a student was greedy if she chose 4 or higher. We will say that a student feels awkward if she knows that the other student knows that she was greedy.

   Assume that $x_1 = 3$ and $x_2 = 5$.
   (a) *6 points.* What is the partition element $P_1(3, 5)$ of student 1? That is, what is the set of values of the pair (x_1, x_2) that is consistent with the information that she knows?

   (b) *6 points.* What is the partition element $P_2(3, 5)$ of student 2?

   (c) *6 points.* Does student 1 know that student 2 was greedy?

   (d) *6 points.* Does student 2 feel awkward?

(e) *6 points.* Does student 2 know that student 1 was not greedy?

(5) *Bonus question.* Choose an integer between 0 and 10. Points will be given as described above.