

PS/EC 172  
MIDTERM EXAM

Please take a three hours stretch of your choice to complete this exam. Collaboration is not allowed. You may consult the lecture notes, your own notes, any textbook and any (inanimate) online resource, but not solutions to previous problem sets or solutions to exams from past years. There is no need to prove any statements that you make. The exam is due in class on Thursday, November 8<sup>th</sup>.

A class of  $n$  students are taking an exam. In one of the questions the students are instructed to write an integer between 1 and 10. That is, each student  $i$  writes a number  $x_i \in \{1, 2, \dots, 10\}$ . The numbers  $\{x_i\}_{i=1}^n$  that the students wrote are collected, and the sum  $s = \sum_i x_i$  is calculated. If  $s \geq 4 \cdot n$ , all the students get no points for that question. If  $n < s < 4 \cdot n$  then each student  $i$  gets  $x_i$  points. If  $s = n$  then all students get 5.5 points. That is:

$$u_i(x_1, \dots, x_n) = \begin{cases} 0 & \text{if } s \geq 4 \cdot n \\ x_i & \text{if } n < s < 4 \cdot n \\ 5.5 & \text{if } s = n. \end{cases}$$

- (1) *25 points.* Assume that  $n = 100$ , and that all students submit their exams simultaneously and without observing the others' answers. Calculate all possible values of  $s$  that are attained in pure Nash equilibria.
- (2) Assume now that  $n = 2$ , and that, as before, both students submit their exams simultaneously and without observing the others' answers.
  - (a) *10 points.* Consider the mixed strategy profile in which each student chooses 3 with probability  $p$  and chooses 4 with probability  $1 - p$ . For which  $p$  is this a mixed Nash equilibrium?
  - (b) *10 points.* Find a correlated equilibrium in which the total expected number of points is higher than in any pure equilibrium (here there is no restriction to 3 and 4; the students can use any actions in  $\{1, 2, \dots, 10\}$ ).
- (3) *25 points.* Assume again that  $n = 2$ , but that now student 1 submits the exam before student 2, and that student 2 knows what student 1 submitted before making her choice; this is thus an extensive form game with perfect information in which student 1 plays before student 2. Find a pure Nash equilibrium of this game which is not subgame perfect.
- (4) Assume again that  $n = 2$ . The students complete the exam without observing each other's choices. When they get their exams back they learn whether they got points for that question.
 

Formally, the students' knowledge space after getting their exam back is  $\Omega = \{1, \dots, 10\}^2$ . At  $(x_1, x_2) \in \Omega$  student 1 knows  $x_1$ , and student 2 knows  $x_2$ , and additionally both know whether or not  $x_1 + x_2 < 8$ .

We will say that a student was greedy if she chose 4 or higher. We will say that a student feels awkward if she knows that the other student knows that she was greedy.

Assume that  $x_1 = 3$  and  $x_2 = 5$ .

  - (a) *6 points.* What is the partition element  $P_1(3, 5)$  of student 1? That is, what is the set of values of the pair  $(x_1, x_2)$  that is consistent with the information that she knows?
  - (b) *6 points.* What is the partition element  $P_2(3, 5)$  of student 2?
  - (c) *6 points.* Does student 1 know that student 2 was greedy?

- (d) *6 points.* Does student 2 feel awkward?
  - (e) *6 points.* Does student 2 know that student 1 was not greedy?
- (5) *Bonus question.* Choose an integer between 1 and 10. Points will be given as described above.