PS/EC 172, SET 3
DUE FRIDAY, APRIL 24TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Elimination of weakly dominated strategies.* In this problem we will show that eliminating weakly dominated strategies can change the set of pure Nash equilibria. This is in contrast to what happens when eliminating strictly dominated strategies, which does not change the set of pure equilibria (see Theorem 2.9 in the lecture notes).

In the following game the additional strategy A was added to matching pennies.

	H	T	A
H	$1, 0$	$0, 1$	$2, 0$
T	$0, 1$	$1, 0$	$1, 0$
A	$1/2, 0$	$0, 1$	$2, 2$

(a) *5 points.* Show that this game has a pure Nash equilibrium.

(b) *5 points.* What are the weakly dominated strategies?

(c) *10 points.* Iteratively remove the weakly dominated strategies. What is the resulting game? What are its pure Nash equilibria?

(2) *Mixed Nash equilibria.* In the *auditing game* a taxpayer has to decide whether to cheat, and the IRS has to decide whether to audit. The benefit to the taxpayer from cheating is some $b > 0$. The cost of auditing is $c > 0$. The fine for cheaters is $f > 0$. Thus the game is described by

	audit	not audit
cheat	$-f, f - c$	$b, 0$
not cheat	$0, -c$	$0, 0$

(a) *20 points.* For every possible value of b, c and f, find all the mixed Nash equilibria.

(b) *20 points.* In what direction does the equilibrium probability of an audit change as a function of b, c and f? How about the probability of cheating?

(c) *20 points.* In what direction do the players' equilibrium expected utilities change as a function of b, c and f?

(3) *The surprise quiz.* A teacher and a student play the following game. The teacher gives a surprise quiz on one of the five days of the work week. The student, who does not know the material, will fail if he does not review the material right before the quiz, but only has time to study on one day. Thus

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each player's set of the strategies is the set of five days of the work week. The student's utility is one if he and the teacher chose the same day, and zero otherwise. The teacher's utility is one minus the student's.

(a) *10 points.* Show that this game does not have a pure Nash equilibrium.

(b) *10 points.* Find a mixed Nash equilibrium for this game.

(c) *Bonus question (1 point).* Show that if there are infinitely many days then there does not exist a mixed Nash equilibrium. Why does this not violate Nash's Theorem?

(4) *Bonus question.* A prisoner escapes to the number line. He chooses some $n \in \mathbb{Z}$ to hide on the zeroth day. He also chooses some $k \in \mathbb{Z}$, and every day hides at a number that is k higher than in the previous day. Hence on day $t \in \{0, 1, 2, \ldots\}$ he hides at $n + k \cdot t$.

Every day the detective can check one number and see if the prisoner is there. If he is there, she wins. Otherwise she can check again the next day.

Formally, the game played between the prisoner and the detective is the following. The prisoner's strategy space is $\{(n, k) : n, k \in \mathbb{Z}\}$, and the detective's strategy space is the set of sequences $(a_0, a_1, a_2, \ldots)$ in $\mathbb{Z}$. The detective wins if $a_t = n + k \cdot t$ for some t. The prisoner wins otherwise.

(a) *1 point.* Prove that the detective has a winning strategy.