PS/Ec 172, Set 4
Due Friday, May 1st

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Chicken, mixed and correlated equilibria.* Consider the game of Chicken, as parametrized by a, b, c, with $a, b, c > 0$ and $b > a$:

	Y	D
Y	a, a	$0, b$
D	$b, 0$	$-c, -c$

(a) *20 points.* Explain why every finite game has a correlated equilibrium.

(b) *20 points.* Find all completely mixed Nash equilibria.

(c) *20 points.* For which values of a, b, c does there exist a symmetric correlated equilibrium with total expected utility larger than that of any pure equilibrium? In this game, a correlated equilibrium is symmetric if the probabilities of (Y, D) and (D, Y) are the same.

(2) *40 points.* Construct an example of a knowledge space with two players, a finite set of states of the world, an event A and a state of the world ω such that $\omega \in K_1 A$, $\omega \in K_2 A$, $\omega \in K_1 K_2 A$, $\omega \in K_2 K_1 A$, but $\omega \notin K_1 K_2 K_1 A$.

(3) *Bonus question.*
(a) *1 point.* Play **ski bum** online with up to three other classmates. You can use Piazza to form groups. You have to play at least twice. Please write down the names of the person / people you played with. You can find the rules here: http://tamuz.caltech.edu/other/ski_bum.pdf.

(b) *1 point.* Contribute at least one comment to a Piazza discussion of this game.

(4) *Bonus question.* A prisoner escapes to $\mathbb{Z}^2$ on Sunday. Every day he must move either one up (i.e., add $(0, 1)$ to his location) or one to the right (add $(1, 0)$), except on Saturdays, when he must rest. The detective can, once a day, check one element of $\mathbb{Z}^2$ and see if the prisoner is there. If she finds him then she wins. He wins if she never finds him.

Formally, the prisoner's strategy is an element

$$(z, f) \in \mathbb{Z}^2 \times \{(1, 0), (0, 1), (0, 0)\}^{\mathbb{N}}$$

such that $f(n) = (0, 0)$ whenever $n \equiv 0 \mod 7$, and $f(n) \in \{(1, 0), (0, 1)\}$ otherwise. The detective's strategy is a sequence $\{z_n\}_{n \in \mathbb{N}}$ with $z_n \in \mathbb{Z}^2$.

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The prisoner's current location when using strategy (z, f) is

$$\ell_n = z + \sum_{k=1}^{n-1} f(k).$$

The detective wins if $\ell_n = z_n$ for some n. The prisoner wins otherwise.

(a) *1 point.* Show that the detective has a winning strategy.

(b) *1 point.* Show that if we remove the requirement that the prisoner rests on Saturdays then the detective does not have a winning strategy.