PS/Ec 172, Homework 6
Due Friday, May 22nd

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Bundling.* Andrew walks into a store with the intention of buying a loaf of bread and a stick of butter. His valuations for the two items are chosen independently from the uniform distribution on $[0,1]$. Rebecca, the store owner, has to set the prices. We assume that Andrew will buy for any price that is lower than his valuation.

  (a) *20 points.* Assume first that Rebecca sets a price b_l for the loaf and b_s for the stick. What is her expected revenue, as a function of b_l and b_s?

  (b) *5 points.* What is the maximal expected revenue she can get?

  (c) *20 points.* Rebecca now decides to *bundle*: she sets a price b_b for buying both items together, and does not offer each one of them separately. That is, she offers Andrew to either buy both for b_b, or else get neither. What is her expected revenue, as a function of b_b?

  (d) *5 points.* What is the maximal expected revenue she can get now?

(2) *Repeated prisoner's dilemma.* Let G_0 be the following version of prisoner's dilemma:

	D	C
D	$0,0$	$3,-4$
C	$-4,3$	$2,2$

Let G be the repeated game in which G_0 is played for T periods. The strategy *tit-for-tat* is the strategy in which a player plays C in the first period, and henceforth always plays the same strategy that the other player played in the previous round. Let s be the strategy profile in which both players play tit-for-tat.

  (a) *25 points.* Let $T = 10$, and let the players' utilities be the sum of their stage utilities. Is s an equilibrium?

  (b) *25 points.* Let $T = \infty$, and let the players' utilities be δ-discounting. For which values of δ is s an equilibrium?

(3) *Bonus question: Mind reading (with high probability).* Ali and Fatima play a game. Ali picks a finite subset $F \subset \mathbb{N}$, and Fatima picks an $n \in \mathbb{N}$. Ali wins if $n \in F$, and Fatima wins otherwise.

Before choosing her n, Fatima picks any subset $S \subseteq \mathbb{N}$. For example, S could be the even numbers. Ali reveals to Fatima the intersection $S \cap F$; we assume he does so truthfully. Fatima can now choose her number n. It

Omer Tamuz. Email: tamuz@caltech.edu.

can depend on Ali's answer, but it cannot be in S. She wins if $n \notin F$, and otherwise Ali wins.

Formally, a pure strategy for Ali is a choice of F. A pure strategy for Fatima is a choice of S, plus a function from subsets of S to $\mathbb{N} \setminus S$; this is the function that specifies n given $S \cap F$.

(a) *1 point.* Show that for every pure strategy of Fatima there is a pure strategy of Ali that ensures that he wins, and that for every pure strategy of Ali there is a pure strategy of Fatima that ensures that she wins.

(b) *1 point.* Show that Fatima has a *mixed* strategy (i.e., a randomly picked strategy) such that for *every* F, her probability of winning is at least $1 - 1/2020$.