PS/Ec 172, Homework 7
Due Monday, June 1st

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *A market for lemons.* Consider the following game of incomplete information, which models the sale of a used car by a seller who knows what it is worth, and a buyer that does not.

There are two players: a buyer and a seller. First, nature makes a chance move, choosing whether the car is good (G) or bad (B), each with probability 1/2. Each of these two histories forms an information set for the seller. In both of these sets, she has to choose between charging a high price (H) and a low price (L). The buyer has two information sets: $\{GH, BH\}$ and $\{GL, BL\}$, so that she only observes the prices, and not the quality of the car. In both of these information sets she has to decide whether to purchase (P) or not (N).

The players' utilities are as follows. The utility of both is zero for any history that ends with the buyer not buying (N). The utility of both is 1 in the histories GHP and BLP, i.e., when the good car is sold for the high price and when the bad car is sold for the low price. The utility of the seller is 10 when she sells a bad car for the high price, and -10 when she sells a good car for the low price. The utility of the buyer is -10 when she buys a bad car for the high price, and 10 when she buys a good car for the low price.

(a) *25 points.* Find all the pure equilibria of this game. Is there any equilibrium in which the car is sold when it is good?

(b) *25 points.* Answer the previous question for the modified game in which the buyer also observes the quality of the car, and so there is perfect information.

(c) *25 points.* Find a behavioral (i.e., mixed) equilibrium in which good cars are sold. Note: to specify a behavioral strategy, you need to write the probability that each action is taken at each information set. In equilibrium no deviation (other behavioral strategy) achieves higher expected utility.

(d) *25 points.* What is the highest probability of selling a good car that can be achieved in equilibrium? This probability is the probability of reaching either GHP or GLP.

(2) *Bonus question: The incredible casino.* A casino has a sequence of slot machines $(M_1, M_2, \ldots)$. Each machine requires the gambler to swipe her credit card, and has a single button. After swiping the card and pressing the

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button, machine M_n credits the gambler 1 dollar with probability $1 - 1/n^2$, and otherwise charges her n^2 dollars.

(a) *1 point.* What is the gambler's expected revenue when using machine M_n?

(b) *1 point.* Kim gambles once at each machine, in order: M_1, M_2, M_3, etc. Explain why, with probability one, her revenue will tend to infinity. Hint: use the Borel-Cantelli lemma. You can read about it on Wikipedia: `http://en.wikipedia.org/wiki/Borel-Cantelli_lemma`.