SS 201a, Set 2
Due Friday, October 25th

Collaboration on homework is encouraged, but individually written solutions are required. Please name all collaborators and sources of information on each assignment. Any such named source may be used.

(1) *Cournot competition.* Find a symmetric equilibrium of the Cournot competition game, as described in the exercise in the lecture notes.

(2) *Elimination of dominated strategies.* Prove Theorem 2.14 from the lecture notes.

(3) *Elimination of weakly dominated strategies.* In the following game the additional strategy A was added to matching pennies.

	H	T	A
H	$1,0$	$0,1$	$2,0$
T	$0,1$	$1,0$	$1,0$
A	$1/2,0$	$0,1$	$2,2$

(a) Show that this game has a pure Nash equilibrium.

(b) What are the weakly dominated strategies?

(c) Iteratively remove the weakly dominated strategies. What is the resulting game? Does it have pure Nash equilibria?

(4) *Double Brouwer.* Let $X \subset \mathbb{R}^n$ be compact and convex. Let $S, T \colon X \to X$ be continuous, and assume that they commute: $S \circ T = T \circ S$.

(a) Prove that S has a fixed point, under the assumption that $n = 1$.

(b) Show that if S has a unique fixed point then it is also a fixed point of T.

(c) *Bonus question.* Show that if T is affine then there is an $x \in X$ that is a fixed point of both S and T. Hint: you can use the technique used in the lecture notes to prove Brouwer for affine T.

(d) *Extra bonus question.* Find such X, S, T with the property that no $x \in X$ is a fixed point of both S and T.

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