SS 201a, Set 3
Due Monday, December 12th

Collaboration on homework is encouraged, but individually written solutions are required. Please name all collaborators and sources of information on each assignment. Any such named source may be used.

(1) *Reserve prices.* Jake and Po Hyun would both like to buy an item owned by Sean. Jake and Po Hyun's valuations are chosen independently from the uniform distribution on $[0, 1]$, and each is known only to her/himself.
   (a) What is Sean's expected revenue from a second price auction?

   (b) Sean now introduces a *reserve price* $b_r \in [0, 1]$: if the maximum bid is under b_r then the auction is canceled, no one gets the item and no one pays. Otherwise, the winner pays the maximum of b_r and the loser's bid. What is Sean's expected revenue, as a function of b_r?

   (c) What is the maximal expected revenue Sean can get by choosing b_r optimally?

(2) *Bundling.* Camila walks into a store with the intention of buying a loaf of bread and a stick of butter. Her valuations for the two items are chosen independently from the uniform distribution on $[0, 1]$. Wenning, the store owner, has to set the prices. We assume that Camila will buy for any price that is lower than her valuation.
   (a) Assume first that Wenning sets a price b_l for the loaf and b_s for the stick. What is her expected revenue, as a function of b_l and b_s?

   (b) What is the maximal expected revenue she can get?

   (c) Wenning now decides to *bundle*: she sets a price b_b for buying both items together, and does not offer each one of them separately. That is, she offers Camila to either buy both for b_b, or else get neither. What is her expected revenue, as a function of b_b?

   (d) What is the maximal expected revenue she can get now?

   (e) *Bonus question.* Assume now that Wenning sets three different prices: b_l for the loaf, b_s for the stick, and b_b for both, so that Camila can choose if to buy just the loaf (for b_l), just the stick (for b_s), or both (for b_b). Assume that Camila will choose to buy whichever items maximize her utility, which is his value for the bought items minus the price paid. What is the maximal expected revenue Wenning can get now?

(3) The Red door has started offering *ratatouille*. Ben and Egor are eager to try it, but both are afraid that it is awful. A-priori, there is a 10% chance that it is awful (A) and a 90% chance that it is good (G).

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There are time periods $t \in \{0, 1, 2, \ldots\}$, and in each time period they each have to simultaneously decide whether to eat (E) it or not (N). Once one of them has decided to eat it, the quality of the ratatouille is revealed to both and never changes.

The stage utility (at any period t) for taking action a with ratatouille of quality q is

$$u_t(a, q) = \begin{cases} 0 & \text{if } a = N \\ 1 & \text{if } a = E \text{ and } q = G \cdot \\ -40 & \text{if } a = E \text{ and } q = A \end{cases}$$

A player's total utility in the game is

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t u_t$$

for $\delta = 9/10$.

(a) Explain why there are no equilibria in which neither of the players ever eat.

(b) Find a pure Nash equilibrium in which Ben's expected utility is higher than Egor's.

(c) Find a symmetric mixed Nash equilibrium.

(d) Find a symmetric correlated equilibrium in which the players' expected utilities are higher than in the symmetric mixed Nash equilibrium.