

SS 201A, SET 3
DUE FRIDAY, NOVEMBER 1ST

Collaboration on homework is encouraged, but individually written solutions are required. Please name all collaborators and sources of information on each assignment. Any such named source may be used.

- (1) *Symmetric equilibria.* A normal form game with two players $G = (\{1, 2\}, (S_1, S_2), (u_1, u_2))$ is *symmetric* if $S_1 = S_2$ and for every s_1, s_2 it holds that $u_1(s_1, s_2) = u_2(s_2, s_1)$. A mixed strategy profile (σ_1, σ_2) is symmetric if $\sigma_1 = \sigma_2$. A correlated equilibrium μ is symmetric if $\mu(s_1, s_2) = \mu(s_2, s_1)$ for all s_1, s_2 .
 - (a) Give an example of a symmetric G that has no symmetric pure equilibria.
 - (b) Prove (without using Brouwer, Nash etc) that every symmetric G has a symmetric mixed equilibrium. Assume $|S_1| = |S_2| = 2$.
 - (c) Show that in a symmetric game there is always a symmetric correlated equilibrium among the correlated equilibria that maximize the sum of expected utilities
- (2) *The surprise quiz.* A teacher and a student play the following game. The teacher gives a surprise quiz on one of the five days of the work week. The student, who does not know the material, will fail if he does not review the material right before the quiz, but only has time to study on one day. Thus each player's set of the strategies is the set of five days of the work week. The student's utility is one if he and the teacher chose the same day, and zero otherwise. The teacher's utility is one minus the student's.
 - (a) Show that this game does not have a pure Nash equilibrium.
 - (b) Find a mixed equilibrium for this game.
 - (c) *Bonus question:* Show that if there are infinitely many days then there does not exist a mixed equilibrium.
- (3) *Very weakly dominant strategies.* Say that a strategy s_i in a normal form game is *very weakly dominant* if for any t_i and any s_{-i} it holds that $u_i(s_{-i}, s_i) \geq u_i(s_{-i}, t_i)$.
 - (a) Find a two player game with a very weakly dominant strategy and an equilibrium in which this strategy is not played.
 - (b) Explain why every finite two player game that has a very weakly dominant strategy has a pure equilibrium in which this strategy is played.
- (4) *Bonus question: Prisoner's Escape.* A prisoner escapes to the number line. He chooses some $n \in \mathbb{Z}$ to hide on the zeroth day. He also chooses some $k \in \mathbb{Z}$, and every day hides at a number that is k higher than in the previous day. Hence on day $t \in \{0, 1, 2, \dots\}$ he hides at $n + k \cdot t$.

Every day the detective can check one number and see if the prisoner is there. If he is there, she wins. Otherwise she can check again the next day.

Formally, the game played between the prisoner and the detective is the following. The prisoner's strategy space is $\{(n, k) : n, k \in \mathbb{Z}\}$, and the detective's strategy space is the set of sequences (a_0, a_1, a_2, \dots) in \mathbb{Z} . The detective wins if $a_t = n + k \cdot t$ for some t . The prisoner wins otherwise.

Prove that the detective has a winning strategy.