SS 201a, Set 4
Due Friday, November 15th

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Equilibria.*

    (a) Let G be a finite extensive form game with perfect information in which each player plays at one history only. That is, for each i there is a single history $h \in H$ such that $P(h) = i$.
Prove: every pure trembling hand perfect equilibrium of the strategic form of G is a subgame perfect equilibrium of G.

    (b) Prove or disprove: every pure subgame perfect equilibrium of a finite extensive form game with perfect information is a trembling hand perfect equilibrium of the strategic form of the game.

(2) Construct an example of a knowledge space with two players, a finite set of states of the world, an event A and a state of the world ω such that $\omega \in K_1 A$, $\omega \in K_2 A$, $\omega \in K_1 K_2 A$, $\omega \in K_2 K_1 A$, but $\omega \notin K_1 K_2 K_1 A$.

(3) *Bonus question.* A prisoner escapes to $\mathbb{Z}^2$ on Sunday. Every day he must move either one up (i.e., add $(0, 1)$ to his location) or one to the right (add $(1, 0)$), except on Saturdays, when he must rest. The detective can, once a day, check one element of $\mathbb{Z}^2$ and see if the prisoner is there. If she finds him then she wins. He wins if she never finds him.

Formally, the prisoner's strategy is an element

$$(z, f) \in \mathbb{Z}^2 \times \{(1,0), (0,1), (0,0)\}^{\mathbb{N}}$$

such that $f(n) = (0,0)$ whenever $n \equiv 0 \mod 7$, and $f(n) \in \{(1,0), (0,1)\}$ otherwise. The detective's strategy is a sequence $\{z_n\}_{n \in \mathbb{N}}$ with $z_n \in \mathbb{Z}^2$.

The prisoner's current location when using strategy (z, f) is

$$\ell_n = z + \sum_{k=1}^{n-1} f(k).$$

The detective wins if $\ell_n = z_n$ for some n. The prisoner wins otherwise.

    (a) Show that the detective has a winning strategy.

    (b) Show that if we remove the requirement that the prisoner rests on Saturdays then the detective does not have a winning strategy.

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