

SS 201A, SET 4
DUE WEDNESDAY, OCTOBER 31ST

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

- (1) Construct an example of a knowledge space with two players, a finite set of states of the world, an event A and a state of the world ω such that $\omega \in K_1A$, $\omega \in K_2A$, $\omega \in K_1K_2A$, $\omega \in K_2K_1A$, but $\omega \notin K_1K_2K_1A$.
- (2) Consider a finite knowledge space. Prove that the collection of connected components $\{C(\omega)\}_{\omega \in \Omega}$ of the graph G_C is the partition that generates the common knowledge algebra Σ_C . (Hint: you can use anything that is proved in the lecture notes.)
- (3) *Bonus question.* A prisoner escapes to \mathbb{Z}^2 on Sunday. Every day he must move either one up (i.e., add $(0, 1)$ to his location) or one to the right (add $(1, 0)$), except on Saturdays, when he must rest. The detective can, once a day, check one element of \mathbb{Z}^2 and see if the prisoner is there. If she finds him then she wins. He wins if she never finds him.

Formally, the prisoner's strategy is an element

$$(z, f) \in \mathbb{Z}^2 \times \{(1, 0), (0, 1), (0, 0)\}^{\mathbb{N}}$$

such that $f(n) = (0, 0)$ whenever $n \equiv 0 \pmod{7}$, and $f(n) \in \{(1, 0), (0, 1)\}$ otherwise. The detective's strategy is a sequence $\{z_n\}_{n \in \mathbb{N}}$ with $z_n \in \mathbb{Z}^2$.

The prisoner's current location when using strategy (z, f) is

$$\ell_n = z + \sum_{k=1}^{n-1} f(k).$$

The detective wins if $\ell_n = z_n$ for some n . The prisoner wins otherwise.

- (a) Show that the detective has a winning strategy.
- (b) Show that if we remove the requirement that the prisoner rests on Saturdays then the detective does not have a winning strategy.