SS 201B, Homework 7
Due Tuesday, March 7th

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

(1) *Train inspections.* In the lecture notes is described an equilibrium of the train inspection game in which poor passengers ride, and are indifferent between paying and not paying.
   (a) Find an equilibrium in which the rich passengers are indifferent in the same way, or show that none exist (or show that one exists for some values of the parameters).

(2) *A market for lemons.* Alex is shopping for a used car in Bob's used car lot. Every car that arrives at the lot is with probability one half in bad condition (worth $1,000), and with probability one half in good condition (worth $3,000). Bob observes the condition of the car, but Alex does not. Bob sets a price for the car (an integral number of dollars between $0 and $10,000), and Alex learns this price and decides whether or not to buy.

   If Alex decides to buy, his utility is the value of the car minus the price. Otherwise his utility is zero. Bob's utility is explained below.
   (a) Draw the tree of this extensive form game. That is, draw the graph whose vertices are the histories and whose edges correspond to possible actions. What are Alex's information sets?

   (b) Assume first that Bob's utility, if Alex buys, is the price, minus the value of the car, plus $100. If no trade occurs her utility is zero (this describes a situation in which Bob buys the cars for a $100 discount). Construct a pure equilibrium in which good cars are sold with positive probability, or explain why no such equilibria exist.

   (c) Assume now that Bob's utility is $1 for a sale at or above the car's worth, -$1 for a sale below the car's worth, and $0 if there is no sale (this describes a situation in which Bob earns a fixed commission per sale).
   Construct a pure equilibrium in which good cars are sold with positive probability, or explain why no such equilibria exist.

(3) *Strategic equivalence.* We say that two normal form games with complete information $G = (N, \{S_i\}, \{u_i\})$ and $G' = (N', \{S'_i\}, \{u'_i\})$ are equivalent if (1) $N' = N$ and $S'_i = S_i$ for all i and (2) for each player i there is a function $f_i \colon S_{-i} \to \mathbb{R}$ such that $u'_i(s_{-i}, s_i) = u_i(s_{-i}, s_i) + f_i(s_{-i})$.
   (a) Show that there is a natural bijection between (mixed) Nash equilibria of G and of G'.

   (b) *Bonus question.* Do the same for correlated equilibria.

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