$\begin{array}{c} \text{SS 205b, Set 2} \\ \text{Due Friday, January 28}^{\text{St}} \end{array}$

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Note: some of the claims that you are asked to prove are false. For these claims please provide a counterexample.

(1) Let $(X_i)_i$, $(\leq_i)_i$ and $(e_i)_i$ be the consumption sets, preferences and endowments of an exchange economy. Suppose that each X_i is closed and that each \succ_i is closed and LNS.

Recall that $(x_i)_i$ and p form a competitive equilibrium if $x_i \in X_i^*(p, p \cdot e_i)$ for all i, and if $\sum_i x_i = \sum_i e_i$.

Recall also that $(x_i)_i$ is Pareto optimal if $x_i \in X_i$ for all i, and if for every $(x'_i)_i$ such that $\sum_i x'_i = \sum_i e_i$ and $x'_i \succeq_i x_i$ for all i it holds that $x_i \succeq x'_i$ for all i.

- (a) Prove that if $(x_i)_i$ and p form an equilibrium then $(x_i)_i$ is Pareto optimal.
- (b) Prove that if (x_i)_i is Pareto optimal then there exists a p such that (x_i)_i and p form an equilibrium.
- (2) Recall that the support function of a compact convex $Y \subset \mathbb{R}^n$ is given by $v_Y(p) = \max_{y \in Y} p \cdot y$.
 - (a) Prove that $Y' \subseteq Y$ iff $v_{Y'}(p) \leq V_Y(p)$ for all p.
 - (b) For $\lambda > 0$ and $Y \subseteq \mathbb{R}^n$ denote $\lambda Y = \{\lambda y, y \in Y\}$. Prove that $v_{\lambda Y}(p) = \lambda v(p)$.
- (3) Consider a private ownership economy where each \leq_i is LNS and convex, and each X_i and Y_j is convex. Show that every Walrasian quasi-equilibrium (as defined in §5 of the lecture notes) is an equilibrium.

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