SS 205B, SET 2

DUE MONDAY, FEBRUARY 7TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Note: some of the claims that you are asked to prove are false. For these claims please provide a counterexample.

(1) Find a compact convex $A \subset \mathbb{R}^n$ (for an n of your choice) and a nonempty, upper-hemicontinuous correspondence $\Gamma\colon A \to A$ that does not have a fixed point.

(2) Suppose that $Y \subseteq \mathbb{R}^L$ is convex. Let K be a convex compact subset of $\mathbb{R}^L$, and $\hat{Y} = K \cap Y$. Show that if $\hat{y} \in \hat{Y}^*(p)$ and $\hat{y}$ is in the interior of K then $y \in Y^*(p)$.

(3) Denote $\mathscr{P} = \{(p_1, p_2) : p_1 \geq 0, p_2 \geq 0, p_1 + p_2 = 1\}$. Let $Z^*: \mathscr{P} \to \mathbb{R}^2$ be a continuous function (not a correspondence!) such that $p \cdot Z^*(p) \leq 0$. Prove that there exists a $p \in \mathscr{P}$ such that $Z^*(p) \in \mathbb{R}^2_-$. You may not use any fixed point theorems in this proof. Hint: the intermediate value theorem may be useful.

(4) Suppose $Y \subset \mathbb{R}^L$ is compact. Show that the correspondence $Y^*\colon \mathbb{R}^L \to \mathbb{R}^L$ given by $Y^*(p) = \operatorname{argmax}_y p \cdot y$ is
   (a) Upper-hemicontinuous. I.e., for every $p^n \to p$ and $y^n \to y$ such that $y^n \in Y^*(p^n)$ it holds that $y \in Y^*(p)$.
   (b) Lower-hemicontinuous. I.e., for every $p^n \to p$ and $y \in Y^*(p)$ there is a sequence $y^n \to y$ such that $y^n \in Y^*(p^n)$.

(5) Prove that if $\Gamma\colon A \to B$ and $\Gamma'\colon A \to C$ are nonempty, compact, upper-hemicontinuous correspondences then so is the correspondence $\Gamma + \Gamma'$ from A to $B + C$ that maps $a \in A$ to $\Gamma(a) + \Gamma'(a)$. Here addition is Minkowski addition.

Omer Tamuz. Email: tamuz@caltech.edu.