Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Note: some of the claims that you are asked to prove are false. For these claims please provide a counterexample.

(1) Find a compact convex \( A \subseteq \mathbb{R}^n \) (for an \( n \) of your choice) and a nonempty, upper-hemicontinuous correspondence \( \Gamma: A \rightrightarrows A \) that does not have a fixed point.

(2) Suppose that \( Y \subseteq \mathbb{R}^L \) is convex. Let \( K \) be a convex compact subset of \( \mathbb{R}^L \), and \( \bar{Y} = K \cap Y \). Show that if \( \bar{y} \in \bar{Y}^*(p) \) and \( \bar{y} \) is in the interior of \( K \) then \( y \in Y^*(p) \).

(3) Denote \( \mathcal{P} = \{(p_1, p_2) : p_1 \geq 0, p_2 \geq 0, p_1 + p_2 = 1\} \). Let \( Z^*: \mathcal{P} \to \mathbb{R}^2 \) be a continuous function (not a correspondence!) such that \( p \cdot Z^*(p) \leq 0 \). Prove that there exists a \( p \in \mathcal{P} \) such that \( Z^*(p) \in \mathbb{R}^2 \). You may not use any fixed point theorems in this proof. Hint: the intermediate value theorem may be useful.

(4) Suppose \( Y \subseteq \mathbb{R}^L \) is compact. Show that the correspondence \( Y^*: \mathbb{R}^L \rightrightarrows \mathbb{R}^L \) given by \( Y^*(p) = \arg\max_y p \cdot y \) is
   (a) Upper-hemicontinuous. I.e., for every \( p^n \to p \) and \( y^n \to y \) such that \( y^n \in Y^*(p^n) \) it holds that \( y \in Y^*(p) \).
   (b) Lower-hemicontinuous. I.e., for every \( p^n \to p \) and \( y \in Y^*(p) \) there is a sequence \( y^n \to y \) such that \( y^n \in Y^*(p^n) \).

(5) Prove that if \( \Gamma: A \rightrightarrows B \) and \( \Gamma': A \rightrightarrows C \) are nonempty, compact, upper-hemicontinuous correspondences then so is the correspondence \( \Gamma + \Gamma' \) from \( A \) to \( B + C \) that maps \( a \in A \) to \( \Gamma(a) + \Gamma'(a) \). Here addition is Minkowski addition.