SS 205B, SET 3
DUE MONDAY, MARCH 1ST

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Note: some of the claims that you are asked to prove are false. For these claims please provide a counterexample.

(1) As in the lecture notes, consider an exchange economy with two agents, a single physical commodity and two states. Assume $e_1 = (1,0)$ and $e_2 = (0,1)$. Let the preference of consumer i be given by

$$u_i(x_{i,s_1}, x_{i,s_2}) = \pi_i w_i(x_{i,s_1}) + (1-\pi_i) w_i(x_{i,s_2}),$$

for some strictly concave, stricty increasing, differentiable $w_i : \mathbb{R} \to \mathbb{R}$ and $\pi_i \in [0,1]$. Show that if $\pi_1 > \pi_2$ then in equilibrium $x_{1,s_1} > x_{1,s_2}$ and $x_{2,s_1} < x_{2,s_2}$.

(2) Consider a pari-mutuel setting in which all gamblers have the same belief π with full support. Show that equilibrium prices satisfy $p = \pi$.

(3) Consider a pari-mutuel setting with $I = L = 2$. Suppose $m_1 = m_2$ and $\pi_{1,1} = \pi_{1,2} = 1/2$. Show that $p_1 = p_2 = 1/2$.

(4) Prove Kirchberger's Theorem. You can use the Separating Hyperplane Theorem (which you do not need to prove) and the following lemma, which you do need to prove.

Lemma 1. *Let A and B be finite subsets of $\mathbb{R}^L$. If $\mathrm{Conv}(A) \cap \mathrm{Conv}(B)$ is non-empty, then there exist $A' \subseteq A$ and $B' \subseteq B$ whose union contains at most $L+2$ points, and such that $\mathrm{Conv}(A') \cap \mathrm{Conv}(B')$ is non-empty.*

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