$\frac{\rm SS~205b,~Set~4}{\rm Due~Wednesday,~March~16^{TH}}$

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Note: some of the claims that you are asked to prove are false. For these claims please provide a counterexample.

(1) As in the lecture notes, consider an exchange economy with two agents, a single physical commodity and two states. Assume $e_1 = (1,0)$ and $e_2 = (0,1)$. Let the preference of consumer *i* be given by

$$u_i(x_{i,s_1}, x_{i,s_2}) = \pi_i w_i(x_{i,s_1}) + (1 - \pi_i) w_i(x_{i,s_2})$$

for some strictly concave, strictly increasing, differentiable $w_i : \mathbb{R} \to \mathbb{R}$ and $\pi_i \in [0, 1]$. Show that if $\pi_1 > \pi_2$ then in equilibrium $x_{1,s_1} > x_{1,s_2}$ and $x_{2,s_1} < x_{2,s_2}$.

- (2) Consider a pari-mutual setting with I = L = 2. Suppose $m_1 = m_2$ and $\pi_{1,1} = \pi_{1,2} = 1/2$. Show that $p_1 = p_2 = 1/2$.
- (3) Consider a pari-mutuel setting in which all gamblers have the same belief π with full support. Show that equilibrium prices satisfy $p = \pi$.
- (4) As in the "Asset pricing" lecture in the lecture notes, consider an exchange economy with a single physical good, and S states.

Let $a \in \mathbb{R}^S$ be an asset such that $a_s > 0$ for all s, and $s \neq s'$ implies $a_s \neq a_{s'}$. Show that there exists a complete asset market consisting entirely of options on a.

- (5) As in Theorem 20.1, suppose that $A_{s,j} \ge 0$, and that for all j there is an s such that $A_{s,j} > 0$. For fixed A, show that the set of prices q such that there are no arbitrage opportunities is convex.
- (6) In Theorem 20.1 we assume that $A_{s,j} \ge 0$, and that for all *j* there is an *s* such that $A_{s,j} > 0$. Find a weaker condition under which the theorem still holds.
- (7) Consider a consumer with endowment $e \gg 0 \in \mathbb{R}^S$ and a strictly monotone preference over her consumption $x \in \mathbb{R}^S_+$. Given a market with assets A and prices q, the consumer's problem is to choose a portfolio $z \in \mathbb{R}^J$ to maximize her consumption x = e + Az, subject to $q \cdot z \leq 0$.

Show that the consumer's problem has a solution if and only if there are no arbitrage opportunities.

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