SS 205B, SET 4
DUE FRIDAY, MARCH 19TH

Collaboration on homework is encouraged, but individually written solutions are required. Also, please name all collaborators and sources of information on each assignment; any such named source may be used.

Note: some of the claims that you are asked to prove are false. For these claims please provide a counterexample.

As in the "Asset pricing" lecture in the lecture notes, consider an exchange economy with a single physical good, and S states.

(1) Let $a \in \mathbb{R}^S$ be an asset such that $a_s > 0$ for all s, and $s \neq s'$ implies $a_s \neq a_{s'}$. Show that there exists a complete asset market consisting entirely of options on a.

(2) As in Theorem 20.1, suppose that $A_{s,j} \geq 0$, and that for all j there is an s such that $A_{s,j} > 0$. For fixed A, show that the set of prices q such that there are no arbitrage opportunities is convex.

(3) In Theorem 20.1 we assume that $A_{s,j} \geq 0$, and that for all j there is an s such that $A_{s,j} > 0$. Find a weaker condition under which the theorem still holds.

(4) Consider a consumer with endowment $e \gg 0 \in \mathbb{R}^S$ and a strictly monotone preference over her consumption $x \in \mathbb{R}^S_+$. Given a market with assets A and prices q, the consumer's problem is to choose a portfolio $z \in \mathbb{R}^J$ to maximize her consumption $x = e + Az$, subject to $q \cdot z \leq 0$.

Show that the consumer's problem has a solution if and only if there are no arbitrage opportunities.

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